Monetary Transmission: Are Emerging Market and Low-Income Countries Different? ANNEX*

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The Framework

Diebold and Li (2006) suggested a modification to the Nelson and Siegel exponential component framework to fit yield curves. They use three time-varying parameters, which can be also interpreted as the level, slope, and curvature. These unobserved parameters are identified based on the data and mean square error optimization, after imposing simple structural restrictions. The state-space representation along with the Kalman filtration allows for missing observations.

The yield curve can be described as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \qquad (1)$$

where $y_t(\tau)$ is the yield at time t of a bond with maturity τ. β_{1t}, β_{2t}, and β_{3t} are the time-varying parameters (or factors) and λ are country-specific parameters driving the exponential decay rate. The parameter β_1 can be interpreted as a level shift, as it increases all maturity yields equally. The

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parameter β_2 represents the slope of the yield curve. The loading on this parameter, $\frac{1-\exp^{-\lambda\tau}}{\lambda\tau}$, is between 0 and 1. The parameter β_3 describes the curvature: its loading, $\frac{1-\exp^{-\lambda\tau}}{\lambda\tau} - \exp^{-\lambda_t\tau}$, starts at 0, increases up to a certain maturity, and gradually decays afterward.

CDR adjust the original framework to ensure that there is no arbitrage among different maturities. The arbitrage-free model adds extra slope and curvature terms to the original model. The arbitrage-free model is as follows:

$$\begin{aligned} y_t(\tau) &= \beta_{1t} + \beta_{2t}^1 \left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_{2t}^2 \left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau}\right) + \cdots \\ \cdots &+ \beta_{3t}^1 \left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_{3t}^2 \left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right), \end{aligned} \quad (2)$$

where β_{2t}^1 and β_{2t}^2 are the two time-varying slope factors and β_{3t}^1 and β_{1t}^2 are the two time varying curvature factors. The slope and curvature factors differ in their lambda parameters. We follow CDR in setting the lambdas: λ_1 is equal to 0.85 and λ_2 is equal to 0.1, implying that the first curvature factor peaks near to the 2-year maturity and the second curvature factor reaches its maximum around the 15-year maturity.

In order to identify unobserved time-varying parameters, we transformed the model to a state-space form. The transition equations driving the dynamics of yields are:

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = A \begin{bmatrix} \beta_{1t} \\ \beta_{2t}^1 \\ \beta_{2t}^2 \\ \beta_{3t}^1 \\ \beta_{3t}^2 \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix}, \quad (3)$$

where

$$A = \begin{bmatrix} 1 & \frac{1-e^{-\lambda_1\tau_1}}{\lambda_1\tau_1} & \frac{1-e^{-\lambda_2\tau_1}}{\lambda_2\tau_1} & \frac{1-e^{-\lambda_1\tau_1}}{\lambda_1\tau_1} - e^{-\lambda_1\tau_1} & \frac{1-e^{-\lambda_2\tau_1}}{\lambda_2\tau_1} - e^{-\lambda_2\tau_1} \\ 1 & \frac{1-e^{-\lambda_1\tau_2}}{\lambda_1\tau_2} & \frac{1-e^{-\lambda_2\tau_2}}{\lambda_2\tau_2} & \frac{1-e^{-\lambda_1\tau_2}}{\lambda_1\tau_2} - e^{-\lambda_1\tau_2} & \frac{1-e^{-\lambda_2\tau_2}}{\lambda_2\tau_2} - e^{-\lambda_2\tau_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_1\tau_N}}{\lambda_1\tau_N} & \frac{1-e^{-\lambda_2\tau_N}}{\lambda_2\tau_N} & \frac{1-e^{-\lambda_1\tau_N}}{\lambda_1\tau_N} - e^{-\lambda_1\tau_N} & \frac{1-e^{-\lambda_2\tau_N}}{\lambda_2\tau_N} - e^{-\lambda_2\tau_N} \end{bmatrix}. \quad (4)$$

The factors, β_i, are assumed to be random-walk processes:

$$\begin{bmatrix} \beta_{1t} \\ \beta_{2t}^1 \\ \beta_{2t}^2 \\ \beta_{3t}^1 \\ \beta_{3t}^2 \end{bmatrix} = \begin{bmatrix} \beta_{1t-1} \\ \beta_{2t-1}^1 \\ \beta_{2t-1}^2 \\ \beta_{3t-1}^1 \\ \beta_{3t-1}^2 \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \\ \eta_{5t} \end{bmatrix} \tag{5}$$

where ε and η are white noise shocks with zero means and covariance matrices Q and H:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \backsim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right]$$

The measurement equations then link the observed yields with the state variables assuming no measurement errors:

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} y_t^{obs}(\tau_1) \\ y_t^{obs}(\tau_2) \\ \vdots \\ y_t^{obs}(\tau_N) \end{bmatrix}. \tag{6}$$

We match the state-space model with the data using the Kalman filter. For each country we estimate matrices Q and H using the Bayesian estimation techniques with the inverse-gamma distribution of priors.

We simplify the CDR framework in three respects, without losing any of its structural advantages. First, we reduce the number of estimated parameters by filtering the noise in the data via the error terms, ε, rather than by treating measurement errors explicitly. Second, we impose random walk processes for the latent factors. Third, we do not allow for cross-factor dynamics and correlations. The last two simplifications follow Diebold, Rudebusch, and Aruoba (2006), who found the factors to be highly persistent with insignificant cross-factor dynamics.

Finally, we merge the partial βs for the slope and curvature to make them comparable with the LS approach. We define the comprehensive slope β_{2t} as a sum of partial slope estimates, that is, $\beta_{2t} = \beta_{2t}^1 + \beta_{2t}^2$; similarly for the curvature β_{3t}.