THE TEACHING MODEL

Aggregate Demand

$$\hat{y}_t = a_1 \hat{y}_{t-1} - a_2 mci_t + a_3 \hat{y}_t^* + \varepsilon_t^y$$

$$mci_t = a_4 \hat{z}_t + (1 - a_4)\hat{r}_t$$

Aggregate Supply

$$\pi_t = b_1 \pi_{t-1} + (1 - b_1)\pi_{t+1}^e + b_2 rmc_t + \varepsilon_t^\pi$$

$$rmc_t = b_3 \hat{y}_t + (1 - b_3)\hat{z}_t$$

Uncovered Interest Rate Parity

$$s_t = s_{t+1}^e + (i_t^* - i_t + prem_t)/4 + \varepsilon_t^s$$

Policy Rule

$$i_t = f_1 i_{t-1} + (1 - f_1)(\pi_{t+1}^e + \tilde{r}_t + f_2(\pi_{t+4} - \pi_{t+4}^T) + f_3 \hat{y}_t) + \varepsilon_t^i$$

This is a reduced-form new-Keynesian model, more complicated versions of which are employed in many central banks. The model consists of four behavioral equations (aggregate demand and supply, uncovered interest rate parity, and monetary policy rule) and several identities. See www.ales-bulir.wbs.cz for the Matlab code and installation instructions.

Aggregate demand

The aggregate spending relationship corresponds to the open economy version of the traditional IS curve and takes the form:

$$\hat{y}_t = a_1 \hat{y}_{t-1} - a_2 mci_t + a_3 \hat{y}_t^* + \varepsilon_t^y, \tag{1}$$

where $\hat{y}_t$ is the deviation of the log of output from its noninflationary level, i.e., the output gap; mci_t is the real monetary condition index that is defined as deviations of the long-term real interest rate, $\hat{r}_t$, and real exchange rate, $\hat{z}_t$, from its neutral (noninflationary) level; $\hat{y}_t^*$ is the foreign output gap and ε_t^y is an aggregate demand shock. The coefficients a_1, a_2, and a_3 capture the persistence of output; the impact of monetary conditions on real economic activity; and the impact of foreign environment, respectively.

Aggregate supply

The aggregate supply equation (the Phillips curve) is defined as follows:

$$\pi_t = b_1 \pi_{t-1} + (1 - b_1)\pi_{t+1}^e + b_2 rmc_t + \varepsilon_t^\pi, \tag{2}$$

where π_t is the annualized quarterly change of the consumer price index, i.e., inflation; π^e_{t+1} denotes model-consistent inflation expectations; rmc_t is the gap in firms' real marginal costs; and ε^π_t is an aggregate supply shock.

The supply relationship encompasses multi-period, overlapping nominal contracts of domestic producers as well as importers. The latter is an important feature of small open economies that typically have a powerful exchange rate channel of monetary transmission. That is why we define in (3) the rmc_t as a weighted average of the output gap (domestic producers) and the gap in real exchange rate (importers) with the coefficient $(1-b_3)$ approximating the weight of imported goods in the consumer basket.

$$rmc_t = b_3\hat{y}_t + (1-b_3)\hat{z}_t . \tag{3}$$

It is important to model expectations properly, capturing the behavior of agents, some of which may be forward-looking, using model consistent expectations, π^e_{t+1}, while others are backward-looking. Economic agents who are assumed to be fully forward-looking comprise $1-b_1$ of the population and b_1 agents follow the rule of thumb of past inflation. Another way of thinking about this parameter is the persistence of inflation—the more persistent inflation, the higher is b_1. The coefficient b_2 captures for the influence of the gap in the real marginal costs on inflation (the slope of the Phillips curve) and measures the sacrifice ratio, i.e., how much output will be lost in order to bring inflation down by 1 percentage point.

Uncovered Interest Rate Parity

We capture the relationship with the world by alternative versions of the uncovered interest rate parity condition. In its pure forward-looking version the UIP relates the behavior of domestic and foreign interest rates and the nominal exchange rate:

$$s_t = s^e_{t+1} + (i^*_t - i_t + prem_t)/4 + \varepsilon^s_t , \tag{4}$$

where s_t is the nominal exchange rate; s^e_{t+1} is its model consistent expectations; i_t is the domestic nominal interest rate (annualized); i^*_t is the foreign nominal interest rate (annualized); $prem_t$ is the risk premium; and ε^s_t is an exchange rate shock. The pure version of the UIP generally makes it difficult to match the observed behavior of the economy (looking, say, at impulse responses), because the exchange rate has little persistence. The pure forward-looking element in (4) makes the current exchange rate level immediately adjust to the sum of all future interest rate differentials implied by the model behavior. This is clearly at odds with the observed exchange rates as these adjust more sluggishly.

There are a few alternative ways of increasing the persistence of the exchange rate. One way is by modifying the pure UIP by reducing its forward-looking nature. This approach substitutes the model-consistent exchange rate expectations in (4) by a combination of backward-looking and model-consistent (forward-looking) expectations (denoted as s^e). The approach below allows for a nonzero rate of growth of the exchange rate in the long-run. Equation for exchange rate expectations (s^E) then takes the form:

$$s_{t+1}^{E} = e_1 s_{t+1}^{e} + (1 - e_1)(s_{t-1} + 2\Delta\bar{s}_t)\,, \tag{5}$$

where

$$\Delta\bar{s}_t = \bar{\pi}_t - \bar{\pi}_t^* + \Delta\bar{z}_t\,, \tag{6}$$

and where $\bar{\pi}_t$ is domestic inflation target; $\bar{\pi}_t^*$ is foreign inflation target; and $\Delta\bar{z}_t$ is the trend (long-run) change in the real exchange rate. The coefficient e_1 determines the degree of forward-looking behavior in the financial market and s^E is thus a composite of backward- and forward-looking expectations.

The second element in (5), $(s_{t-1} + 2\Delta\bar{s}_t)$, is the backward-looking exchange rate expectations, which project the exchange rate in period $t + 1$ as an extrapolation of the past ex-change rate using the trend rate of growth of the real exchange rate and the average inflation differential approximated by the difference in inflation targets. While such expectations are not model-consistent in the short-run, they are consistent in the long-run, in line with the finding that the UIP holds at longer horizons only. The term $\Delta\bar{s}_t$ is the change in the exchange rate consistent with long-term economic fundamentals represented by the inflation targets and the real exchange rate trend. By construction, $\Delta\bar{s} = \Delta s$ in the long-run, so the long-run properties of the model are intact. The above extension of the UIP is designed to make the nominal exchange rate more persistent, however, it does not reduce the overall volatility of the exchange rate of the simulated model (the movements of the nominal exchange rate are more persistent, but its standard deviation is not necessarily smaller).

Alternatively, the UIP can be modified for central banks using FOREX interventions actively (and the exchange rate channel) to meet the inflation target. However, to model the exchange rate consistently, the change in the ER target must be defined as a "sum" of the inflation differential and trend appreciation as is in (6). Moreover, in order to avoid counterintuitive exchange rate behavior in the first simulated period, the last observed level of the exchange rate need to be taken as the target at time *t-1*. The targeted exchange rate is then defined as:

$$s_t^T = s_{t-1}^T + \Delta\bar{s}_t / 4\,, \tag{7}$$

$$s_{t-1}^T = s_{t-1}\,, \tag{8}$$

where s_t^T is the exchange rate target at time t. Thus, first, equation (6) determines the long-run change in the nominal exchange rate as implied by PPP theory and makes the *change* in the exchange rate target consistent with the chosen inflation target. Second, the target for the *level* of the nominal exchange rate is determined by its connection to the last observed level of the nominal exchange rate.

The UIP then becomes a "weighted average" of the ER target and the pure UIP

$$s_t = e_1 s_t^T + (1 - e_1)(s_{t+1}^e + (i_t^* - i_t + prem_t)/4) + \varepsilon_t^s\,, \tag{9}$$

with the coefficient e_1 measuring the presence of the central bank on the FOREX market. The extended UIP (9) stabilizes the exchange rate in a way that resembles managed exchange rate regime (Beneš, Hurník, and Vávra, 2008).

The stability of the exchange rate, however, comes at the cost of persistent fluctuations of inflation whenever a shock hits the economy: the inflation forecast oscillates around the target level. Intuitively, the introduction of an exchange rate target is equivalent to switching from an inflation target to a price level target. Even if the central bank publicly defines its target as a rate of growth of the price level, it simply means that the target is a price level growing at some constant rate. Any overshooting of such an inflation target must still be compensated by subsequent undershooting and *vice versa* to jointly satisfy the inflation and exchange rate targets.

The model also satisfies the long-run version of the UIP expressed in real variables. This version of the UIP binds together the trend values for real exchange rate appreciation (either due to the Balassa-Samuelson effect or some "convergence inflation") and the trend values of domestic and foreign real interest rates:

$$\Delta \bar{z}_{t+1} = \bar{r}_t - \bar{r}_t^* - prem_t. \tag{10}$$

The trend values for $\Delta \bar{z}$, $\bar{r}$ and $\bar{r}^*$ are set as parameters in this model, requiring prior assessment of these trend values, while the risk premium is calculated endogenously, assuring the existence of a consistent steady state. Values of those parameters frame the forecast over the medium to long run.

Policy Rule

The model is closed by a policy reaction function of the monetary authority (Taylor, 1993). For simplicity, we take the three-month interest rate to be the instrument of monetary policy, and the authority is assumed to respond to deviations of next-period inflation from its target and to the output gap. In other words, we assume that the credit markets flawlessly transmit the changes in the policy rate into money-market rates. The last-period policy stance may also affect the current policy stance:

$$i_t = f_1 i_{t-1} + (1 - f_1)(i_t^n + f_2(\pi_{t+1}^e - \pi^T) + f_3 \hat{y}_t) + \varepsilon_t^i, \tag{11}$$

where i_t is the domestic short-term nominal interest rate and ε_t^i is a policy shock. The monetary authority is fully forward-looking and uses model-consistent inflation expectations, π_{t+1}^e. The policy-neutral rate, $i_t^n = \tilde{r}_t + \pi_{t+1}^e$, a sum of the trend real interest rate and model-consistent inflation expectations, is such that keeps the output gap unchanged.