A FISCAL MODEL

I. THE MAIN SECTORS

We use a closed-economy, dynamic general equilibrium model (DGE) to simulate alternative fiscal policies. The model is relatively complex, encompassing all the nominal and real rigidities that have been introduced into the DGE models in the academic literature or modeling practice to match the observed data. The model thus encompasses not only the usual price rigidity but also rigidity in nominal wages. In addition it introduces real rigidity in investment and a proportion of households that are liquidity constrained (hand-to-mouth) and whose current consumption depends on current income.

The model is in a non-linear and strictly stationary form, which means that the steady-state values for all variables are constant. None of the model variables, including the price level, are growing in the steady-state. Regarding the non-linearity attribute, it enables an easy move from the solution of the agent's optimization problem to the model code without a need to log-linearize the model. The only exception are the equations for price adjustment (Phillips curves), the log-linear form of which is well known. Regarding the stationarity of the model, it allows us to keep the model tractable.

The model consists of seven basic sectors:

1. Households
2. Intermediate goods producers
3. Consumption goods producers
4. Investment goods producers
5. Government goods producers
6. Government
7. Central bank

A. Households

To ensure non-Ricardian behavior of households requires that the economy is populated by two types of households. While the first household is of an optimizing type, with the resulting "Ricardian behavior," the second one represents a hand-to-mouth consumer that fully consumes the current period income and behaves in a non-Ricardian manner.

Optimizing households

The optimizing households consume final goods, accumulate capital, supply labor services and capital to intermediate producers and participate in the perfectly flexible financial market. In addition they pay wage and consumption taxes and receive government transfers as well as profits of all firms that they own.[1] Optimizing households maximize their lifetime utility derived from consumption and adjusted for specific, empirically-observed habits, subject to budget constraints. The utility function assumes a unit elasticity of intertemporal substitution and is separable in consumption and leisure (1).

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t^{opt} - \chi H_t\right) + \left(1 - L_t\right)\right], \tag{1}$$

where C_t^{opt} is the level of real consumption of optimizing households,[2] H_t is the household "habit persistence," and χ is the weight of past consumption on current consumption decisions, L_t is labor supplied by optimizing households. The household habit persistence H_t is described by equation (2):

$$H_t = C_{t-1} e^{\varepsilon\_habit_t} \tag{2}$$

where C_t is the overall level of real consumption and ε_habit_t is a consumption shock. The habit persistence is determined by total consumption as opposed by that of optimizing households only to capture the so-called "keeping up with the Joneses" behavior referring to the comparison to one's neighbor as a benchmark for the accumulation of material goods. In other words, if optimizing households spend, so do the hand-to-mouth households, too. Overall consumption aggregates the contributions from both the optimizing and hand-to-mouth households:

$$C_t = C_t^{opt} + C_t^{htm}. \tag{3}$$

Optimizing households maximize their lifetime utility function subject to a budget constraint:

$$P_t^C C_t^{opt}(1+\tau_c) + P_t^J J_t + B_t = (1-\tau_w) W_t L_t + P_t^K K_t + B_{t-1} I_t + Tf_t + \Omega_t, \tag{4}$$

where P_t^C is the price of the consumption good and τ_c is the consumption tax rate; J_t and P_t^J are the real investment good and its price, respectively; B_t is the stock of bonds (savings) held by households; W_t is the nominal wage and τ_w is the wage tax rate; K_t and P_t^K are the capital stock and the rental price paid by the intermediate sector firms, respectively; I_t is the nominal interest rate; Tf_t are government transfers; and Ω_t are firm profits.

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[1] Given the Ricardian behavior of optimizing households, government transfers and firm profits enter only the budget constraint and are not a part the households' first-order conditions discussed below.

[2] Strictly speaking, households consume a basket of differentiated consumption goods produced by *n* producers and costlessly aggregated into C_t^{opt} . Similar processes follow for the other baskets, C_t^{rule} , J_t , and G_t .

The first-order conditions of the household optimization problem determine household consumption and labor decisions and are described in equations (5) and (6):

$$\frac{C_{t+1}^{opt} - \chi H_{t+1}}{C_t^{opt} - \chi H_t} = \frac{\beta}{prem_t} e_t^I \frac{P_t^C (1+\tau_t^c)}{P_{t+1}^C (1+\tau_{t+1}^c)}, \tag{5}$$

$$\frac{W_t}{P_t^C (1+\tau_t^c)} = C_t^{opt} - \chi H_t, \tag{6}$$

where β is the discount factor, and *prem* is the so-called risk premium that pushes the real and nominal interest rates below or above the compounding factor, $1/\beta$, depending on whether the net government debt is negative (that is, the government is a net creditor) or positive (the government is a net debtor), respectively. Intuitively, borrowing costs are to be higher in Brazil (a net debtor with the net debt-to-GDP ratio of 45 percent) than in Singapore (a net creditor). (The formula for the risk premium is formally described in the section Budget Accounting, Net Government Debt, and Risk Premium.)

Equation (5) is the so-called Euler equation determining the choice between current and future consumption, ensuring that utility from consumption is the same across all periods when adjusted for the subjective discount factor, the real interest rate, and the change in indirect taxes. Equation (6) describes the household choice between consumption and leisure depending on the real wage net of taxes.

Moreover, households are not allowed to adjust the nominal wage in every period and this relationship in captured in the Phillips curve. In each period only a fraction of households $(1-\xi^W)$ is allowed to reoptimize their wage contract, while the rest follow a simple indexation scheme and adjust their wage contracts for the last observed wage growth, yielding the following wage adjustment mechanism:[3]

$$\pi_t^W = \frac{\beta}{1+\beta}\pi_{t+1}^W + \frac{1}{1+\beta}\pi_{t-1}^W + \frac{(1-\beta\xi^W)(1-\xi^W)}{\xi^W(1+\beta)} \log\left(\frac{W_t^{flex}(1-\tau^W)}{W_t}\lambda^W\right) + \varepsilon^W, \tag{7}$$

where π_t^W is wage inflation, W_t^{flex} is the "flexible" nominal wage (a wage that would have prevailed on the market without any nominal/real distortions and rigidities), W_t is the overall level of nominal wages that enters firms optimization problem, λ^W is the markup over marginal product of labor that prevails on the labor market, and ε^W is a wage cost-push shock.

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[3] The so-called Calvo parameter, $(1-\xi^W)$, is known also as the "Calvo fairy"—imagine that a magical creature touches you and only then you awake from your dream and make your optimization decision. Or imagine the "Calvo devil," who keeps pouring beer to workers until ξ of them are incapable to asking for a pay raise.

Hand-to-mouth households

Beside the optimizing households who work and save, the economy also contains some households that do not work and whose consumption is financed by government transfers:

$$C_t^{htm} P_t^C = Tf_t \,, \tag{8}$$

where C_t^{htm} is the level of real consumption of hand-to-mouth households and Tf_t is the nominal level of government transfers.

Investment

The optimizing households also own all the capital in the economy and make saving decisions based on the return to capital. In order to bring this process closer to the observed persistence in investment, we introduce intertemporal adjustment cost.[4] Without these costs, the DGE models imply unrealistically fast saving/investment decisions. Capital accumulation is described by equation (9) and the optimization problem yields equations (10) – (11):

$$K_t = K_{t-1}^{1-\delta}\left(\frac{J_t e^{\varepsilon\_accum_t}}{\delta}\right)^{\delta} , \tag{9}$$

$$Q_t = \frac{P_t^J J_t}{\delta K_t} , \tag{10}$$

$$Q_t = \left(\frac{P_{t+1}^K + Q_{t+1}(1-\delta)\dfrac{K_{t+1}}{K_t}}{I_t}\right) , \tag{11}$$

where Q_t is the shadow price of capital, δ is the depreciation rate, and ε_accum_t is a shock to capital accumulation. More specifically, equation (10) defines the shadow price of capital as the replacement cost of one unit of depreciated capital and equation (11) describes its intertemporal dynamics in relation to the rental price of capital and cost in terms of the nominal interest rate.

B. Intermediate Good Producers

Producers of intermediate goods hire capital and labor from the pool of optimizing households. They operate on a monopolistically competitive market and each of them produces one type of an intermediate good.[5] Their production technologies are identical,

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[4] Capital accumulation is non-linear, exhibiting decreasing returns to investment, which is equivalent to convex adjustment costs.

[5] The intermediate good is a theoretical concept that is close to economy's value added, but does not have direct observed equivalent in national accounts.

involving labor-augmenting technological progress. The producers are profit maximizers given their technologies, the cost of production factors, and the nonzero probability (ξ^{Y}) that they will be unable to optimize their product price at every period. The differentiated intermediate goods are then combined into a composite intermediate good that is sold to producers of consumption, investment, and government consumption goods.

The production technology is of the Cobb-Douglas type (equation 12) and equations (13) and (14) determine demand for capital and labor, respectively, given the profit maximization assumption, prices, and technology:

$$Y_t = K_{t-1}^{(1-\alpha)} (A_t L_t)^{\alpha} e^{\varepsilon\_y_t}, \tag{12}$$

$$(1-\alpha) Y_t P_t^{Yflex} = K_t P_t^{K}, \tag{13}$$

$$\alpha Y_t P_t^{Yflex} = L_t W_t (1+\tau^{S}), \tag{14}$$

where Y_t is the level of the real intermediate product (output), A_t is the labor augmenting technology, P_t^{Yflex} is the "flexible-price" deflator of the intermediate product (that is, the price level that would have prevailed in the absence of nominal rigidities), α is the labor share in output, τ^{S} is the social and health security tax paid by firms, and ε_y is a temporary productivity shock.

The producers cannot optimize their prices every period, creating a difference between the market-wide and flexible prices of the intermediate product in the Phillips curve (15):

$$\pi_t^{Y} = \frac{\beta}{1+\beta}\pi_{t+1}^{Y} + \frac{1}{1+\beta}\pi_{t-1}^{Y} + \frac{(1-\beta\xi^{Y})(1-\xi^{Y})}{\xi^{Y}(1+\beta)} \log\left(\frac{P_t^{Yflex}}{P_t^{y}}\lambda^{Y}\right) + \varepsilon^{Y} \tag{15}$$

where π_t^{Y} is intermediate product inflation, P_t^{Y} is the market price of the intermediate goods, λ^{Y} is the markup over marginal cost on the intermediate good market, and ε^{Y} is an intermediate price cost-push shock.

C. Consumption Good Producers

Consumption good producers also operate on a monopolistically competitive market and use domestic intermediate goods as production inputs. Prices are sticky and only a fraction $(1-\xi^{C})$ of producers is able to optimize the price each period. For the sake of simplicity, the production technology (16) is trivial, depending only on intermediate goods, and so is the flexible-price equation (17). However, price stickiness creates a difference between the market-wide and flexible prices (18):

$$C_t = Y_t^{C} \tag{16}$$

$$P_t^{Cflex} = P_t^{Y} \tag{17}$$

$$\pi_t^C = \frac{\beta}{1+\beta}\pi_{t+1}^C + \frac{1}{1+\beta}\pi_{t-1}^C + \frac{(1-\beta\xi^C)(1-\xi^C)}{\xi^C(1+\beta)}\log\left(\frac{P_t^{Cflex}}{P_t^C}\lambda^C\right) + \varepsilon^C \tag{18}$$

where Y_t^C is the intermediate product used by consumption good producers, P_t^{Cflex} and P_t^C are the flexible and market prices of consumption goods, respectively, π_t^C is consumption inflation, λ^C is the markup over marginal cost on the consumption goods market, and ε^C is a consumption price cost-push shock.

D. Investment Good Producers

Investment good producers also operate on a monopolistically competitive market and use intermediate goods as the only input, however, this sector is subject to productivity shocks. Only a fraction $(1-\xi^C)$ of these producers are able to optimize their prices each period:

$$J_t = A_t^J Y_t^J \tag{19}$$

$$P_t^{Jflex} = P_t^Y \tag{20}$$

$$\pi_t^J = \frac{\beta}{1+\beta}\pi_{t+1}^J + \frac{1}{1+\beta}\pi_{t-1}^J + \frac{(1-\beta\xi^J)(1-\xi^J)}{\xi^J(1+\beta)}\log\left(\frac{P_t^{Jflex}}{P_t^J}\lambda^J\right) + \varepsilon^J \tag{21}$$

where Y_t^J is the intermediate product used by investment good producers, A_t^J is a sector specific productivity, P_t^{Jflex} is the flexible price of investment goods, π_t^J is the consumption inflation, P_t^J is the market price of investment goods, λ^J is the markup over marginal costs that prevails on the investment good market, and ε^J is the investment price cost-push shock.

E. Government Good Producers

Finally, the government consumption good producers use the domestic intermediate goods for production. For simplicity, their prices are assumed to be as sticky as the prices of the intermediate goods:

$$G_t = Y_t^G \tag{22}$$

$$P_t^{Gflex} = P_t^Y \tag{23}$$

where Y_t^G are the intermediate goods used by the government good producers, and P_t^{Gflex} is the price of government goods.

F. Government

The government collects the following three types of revenue: the wage, consumption, and social and health security taxes. While the base for both the wage and social/health taxes is the nominal wage bill (the wage rate times the hours worked), the base of the consumption tax is nominal consumption ($P^c C$) On the expenditure side, the government makes transfer payments to households and purchases government consumption goods. The government may accumulate debt, but must remain solvent for all future periods. To this end, we apply a simple debt targeting rule that would adjust either revenue or expenditure, or both, ensuring intertemporal solvency of the government. Intuitively, governments vary their primary balance-to-output ratio, Bal_t, to stabilize debt around a level prescribed by, say, a law or a domestic political consensus. Equations (24) – (30) describe the behavior of the government:

$$\frac{Bal_t}{P_t^Y Y_t}\eta_t^G = -(1-e^{I_t})\left(\frac{NGD}{P^Y Y}\right)_{SS} + \varsigma\left[\left(\frac{NGD_t}{P_t^Y Y_t}\right) - \left(\frac{NGD}{P^Y Y}\right)_{SS}\right] \tag{24}$$

$$\eta_t^G = (\eta_{t-1}^G)^{\rho} e^{\varepsilon\_G} \tag{25}$$

$$Tf_t = W_t^{\omega}\eta_t^{Tf} \tag{26}$$

$$\eta_t^{Tf} = (\eta_{t-1}^{Tf})^{\rho} e^{\varepsilon\_Tf} \tag{27}$$

$$\tau_t^S = \tau_{SS}^S e^{\varepsilon\_\tau^S} \tag{28}$$

$$\tau_t^W = \tau_{SS}^W e^{\varepsilon\_\tau^W} \tag{29}$$

$$\tau_t^C = \tau_{SS}^C e^{\varepsilon\_\tau^C} \tag{30}$$

where NGD_t is net government debt, $\left(\frac{NGD}{P^Y Y}\right)_{SS}$ is the steady-state, that is, targeted debt-to-output ratio, ς captures the aggressiveness of fiscal policy (reaction to deviations of the current debt-to-output ratio from its steady-state level), η_t^G and η_t^{Tf} are persistent shocks to government spending on goods and services and to government transfers, respectively, ρ is the persistency parameter of transfers, $\tau_{SS}^S, \tau_{SS}^W, \tau_{SS}^C$ are the steady-state rates for the social and health security, wage, and consumption taxes, respectively, and $\varepsilon\_\tau^G, \varepsilon\_\tau^{Tf}, \varepsilon\_\tau^S, \varepsilon\_\tau^W, \varepsilon\_\tau^C$ are shocks to government spending on goods and services, transfers, and to the social and health security, wage, and consumption taxes, respectively. The long-lasting shocks to government spending on goods and services and transfers were included to capture the persistent spending behavior of the government.

Equation (24) is the debt stabilizing fiscal rule: the government manipulates its primary balance to remain intertemporally solvent. The first part of the rule states that whenever the government is a net debtor, the primary balance must cover the interest payments paid on the accumulated debt. The second part of the rule captures the reaction to any deviation of the debt-to-output ratio from its targeted value—the government adjusts spending on goods and services, transfers, taxes, or all of them. To study the economic effects of alternative forms of fiscal adjustment, however, it is generally much easier to let the burden of adjustment fall on one item only. For example, one can assume that transfers are set to remain stable in proportion to nominal wages (26) and that tax rates are constant (28 - 30), and then only government spending on G_t varies in accordance to the desired primary balance. Equations (25) and (27) introduce persistency in shocks to both types of government expenditures.

G. Monetary Policy

Monetary policy is credible and the central bank follows a simple forward-looking policy rule. To achieve its price stability objective, the bank sets the nominal interest rate, taking into account, first, its past level and, second, deviations of expected inflation from the inflation objective.

$$I_t = \gamma\, I_{t-1} + (1-\gamma)\left[\left(\log\left(\frac{prem_t}{\beta}\right) + \pi_{t+1}^C\right) + \nu\pi_{t+1}^C\right] + \varepsilon^I , \tag{31}$$

where I_t is the domestic short-term nominal interest rate, γ is the parameter of policy persistency, capturing interest rate smoothing, ν is the parameter of policy aggressiveness, capturing inflation aversion, and ε^I is a monetary policy shock. The monetary authority is forward-looking and uses model-consistent inflation expectations of consumption inflation, π_{t+1}^C. The policy-neutral nominal interest rate, $\left(\log\left(\frac{1}{\beta}\, prem_t\right) + \pi_{t+1}^C\right)$, is such that would keep consumption and investment unchanged.

H. Budget Accounting, Net Government Debt, and Risk Premium

The government remains intertemporally solvent, which in this case is equivalent to stationary net government debt. Positive net debt in the steady-state (the government is a net borrower) implies a primary budget surplus to cover the interest payments and thus avoid debt explosion. Equations (32) - (34) capture revenues, expenditures, and the overall balance.

$$Rev_t = W_t L_t(\tau^S + \tau^W) + P_t^C C_t \tau^C \tag{32}$$

$$Exp_t = P_t^{Gflex} G_t + Tf_t \tag{33}$$

$$Bal_t = Rev_t - Exp_t \tag{34}$$

where Rev_t are revenues, Exp_t are expenditures, and Bal_t is the primary budget balance. The latter then determines the path of net debt accumulation, (35):

$$NGD_t = NGD_{t-1} e^{I_t} - Bal_t \tag{35}$$

The level of net government debt affects the interest rate paid to government creditors, that is, the risk premium (36). The risk premium enters both the Euler equation (3) and monetary policy rule (31).

$$prem_t = \left(\frac{NGD_t}{P_t^Y Y_t}\right)^{\mu} \tag{36}$$

where $prem_t$ is the risk premium and μ is the sensitivity parameter of net debt (the larger μ the more will the risk premium react to changes in net debt).

II. Technologies and Identities

There are two technologies in the model, namely the labor-augmenting technology of intermediate good producers and the investment-specific technology of investment good producers. Both technologies are stationary and equal to one in the steady-state, however, their introduction enables to study effects of permanent technology shocks, equations (37) and (38):

$$A_t = A_{t-1}^{\theta} e^{\varepsilon\_A} \tag{37}$$

$$Aj_t = Aj_{t-1}^{\theta} e^{\varepsilon\_Aj} \tag{38}$$

where ε_A and ε_Aj are permanent shocks to technologies of intermediate good and investment good producers and θ is the persistency parameter of shocks to technologies.

We close the model with an identity that states that intermediate output must be fully used as input for production of consumption, investment, and government consumption goods, respectively, equation (39).

$$Y_t \equiv Y_t^C + Y_t^J + Y_t^G \tag{39}$$

III. Model Calibration

The baseline version of the model is calibrated using parameters that loosely reflect the situation in the Czech economy before the 2008-2009 crisis. We split the calibration process into the calibration of parameters that influence the steady-state of the model and parameters that determine the business-cycle properties of the model.

A. Steady-State Parameters

The model converges to its steady-state values in each of its variables and these stay constant, that is, the model is strictly stationary. For example, the price level is assumed to be constant in the steady-state, implying zero inflation; the capital/output ratio is constant, implying a constant rate of growth of output. Therefore, steady-state parameters are those that impact the level of consumption, investment, output, and so on (Table 1).

Table 1. Steady-State Structural Parameters

Parameter	Description	Calibrated value
β	Household discount factor	0.99
α	Share of labor in intermediate production	0.5
δ	Depreciation rate	0.05
λ^w	Mark-up: nominal wage	1.3
λ^c	Mark-up: consumption good producers	1.3
λ^j	Mark-up: investment good producers	1.3
λ^y	Mark-up: intermediate good producers	1.3
μ	Elasticity of risk premium to net government debt	0.01
ω	Ratio of government transfers to nominal wages	0.8
$\left(\frac{NGD}{P^Y Y}\right)_{SS}$	Government debt-to-output ratio	0.5
τ_{SS}^{C}	Consumption tax rate	0.2
τ_{SS}^{W}	Wage tax rate	0.2
τ_{SS}^{S}	Social and health security tax rate	0.2

While these parameters can be changed, it should be done only with extreme caution as these changes would have nontrivial effects on the model solution. In particular, an increase in the debt-to-output ratio would affect negatively the steady-state values of capital and output: the higher is the debt-to-output ratio, the higher is the risk premium, the real interest rate, and, therefore, the lower is the level of capital and output.

B. Business-Cycle Properties

Business cycle properties of the model, that is, its dynamics around the steady-state, are affected by parameters presented in Table 2.

Table 2. Structural Parameters with No Impact on the Steady State

Parameter	Description	Calibrated value
χ	Households habit persistence	0.85
ξ^w	Calvo parameter: nominal wages	0.8
ξ^c	Calvo parameter: consumption good producers	0.7
ξ^y	Calvo parameter: intermediate good producers	0.4
ξ^j	Calvo parameter: investment good producers	0.25
γ	Monetary policy persistency	0.5
ν	Monetary policy aggressiveness	1.5
ς	Fiscal policy aggressiveness	1.5
ρ	Persistency of government spending on goods and services and transfers	0.6
θ	Persistency of shocks to technologies	0.6

The above parameters can be calibrated more freely. Table 3 discusses some appropriate values for these coefficients as well as their impact on model properties.

Table 3 Appropriate Parameter Values and Other Suggestions

χ	**Households habit persistence** varies between 0 (no persistence) to 0.9 (high persistence). The higher the parameter the less is the current-period consumption affected by shocks to consumption preferences and to government transfers.
ξ^w	**Calvo parameter for nominal wages**. The value varies between 0 (flexible wages) to 0.8 (highly rigid wages); the higher the parameter the more sluggish is the wage adjustment in response to economic developments. *Calibration:* The term $\frac{1}{1-\xi^w}$ denotes the frequency of wage adjustments. For example, calibrating ξ^w equal to 0.8 implies that nominal wages are on average adjusted every 5 periods. It is best to base the calibration on relevant microeconomic studies.
ξ^c	**Calvo parameter for consumption prices**. The value varies between 0 (flexible prices) and 0.8 (highly rigid prices); the higher the parameter the more sluggish is the price adjustment. *Calibration:* Same rule as for wages applies.
ξ^y	**Calvo parameter for intermediate prices**. Same as above.
ξ^j	**Calvo parameter for investment prices**. Same as above.
γ	**Policy rate persistence in the Taylor rule**. The value varies between zero (no persistence in policy setting) to 0.8 (“wait-and-see” policy). The linear homogeneity condition applies: $0<\gamma<1$, otherwise the model becomes explosive.
ν	**Monetary policy aggressiveness**. The weight the policy maker puts on stabilizing inflation.

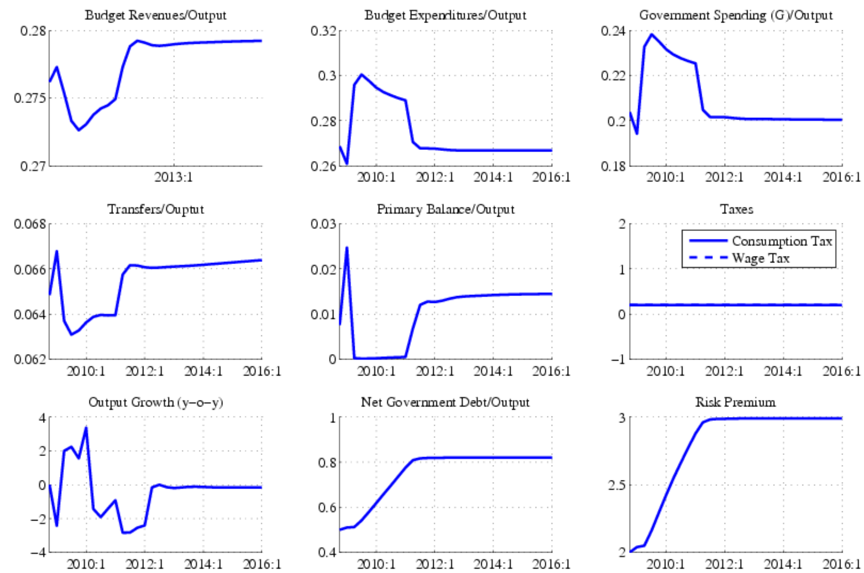
	Behind this calibration is a social loss function of the form $L = \nu\pi^2 + (y - y^{trend})^2$. Value of the parameter typically varies between 0.5 and 2. The so-called Taylor principle applies: ν >0, otherwise monetary policy does not play its stabilization role.
ς	**Fiscal policy aggressiveness**. The government adjusts its primary budget balance to remain intertemporally solvent. Values larger than one (ς >1) stabilize government debt.
ρ	**Persistency of government expenditures.** This parameter introduces persistency in government expenditures (spending on goods and services and transfers) in case a shock to expenditures occurs. "Slowing down" the model's speed of adjustment is useful for simulation purposes and its value ranges from 0 to 0.9.
θ	**Persistency of shocks to technologies.** This parameter introduces persistency in shocks to technologies. It is useful for simulation purposes and its value ranges from 0 to 0.9.

IV. Model Output

Each model scenario produces a two-page output with a total of 18 charts. The example below shows a debt-financed fiscal stimulus through a temporary increase in government spending. In each quarter the stock of debt increases by 4 percent of GDP and this expansion is set to continue for 2 years.

Report on Permanent Increase in the Debt-to-Output Ratio 2010-02-14 06:37:52

Debt-Financed Increase in Government Spending on Goods and Services(G)



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